MATH 221: Midterm 2

Name: key

Directions: No technology, internet, or notes. **Simplify all expression for full credit**. If you have a question, ask me. Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
		50

1. Short answer questions. Fully justify each answer.

   (a) When demand for a product is inelastic and the price is increased, why does revenue increase?

   If we consider revenue as a function of price, we saw that $R'(p) = f(p)\left(1 - E(p)\right)$. inelastic demand is $E(p) < 1$.

   This means $R'(p) = + \cdot + = +$. Since $R'(p) > 0$, $R(p)$ is increasing.

   (b) True or False: If $I(t)$ is the consumer price index, then $I''(t)$ tells us the inflation rate. If not, what does $I''(t)$ tell us?

   False, $I''(t)$ tells us the rate at which inflation is growing or slowing down.

   (c) If $s(t)$ is a position function, why does $s'(t)$ tell us the velocity at a certain time point?

   The derivative of $s(t)$ tells us the instantaneous rate of change of position. By definition this is velocity.

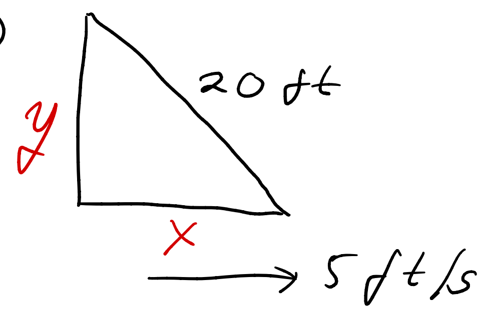
   (d) In related rates problems, you can plug in givens whenever you want. If not, when can you plug them in?

   False; plug them in after step 5/after applying $\dfrac{d}{dt}$ on both sides.

2. A 20 foot ladder leaning against a wall begins to slide. How fast is the top of the ladder sliding down the wall at the instant of time when the bottom of the ladder is 12 feet away from the wall and sliding away from the wall at a rate of 5 feet/second?

   Hint: pythagorean theorem.

①



② given

$x = 12$

$\frac{dx}{dt} = 5$

unknown

$\frac{dy}{dt}$

③ $x^2 + y^2 = 20^2$ ← The length of a ladder is NOT a function of time. Only write variables over quantities which change over time.

④ $\frac{d}{dt}x^2 + \frac{d}{dt}y^2 = \frac{d}{dt}400$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

need to eliminate in ⑤

⑤ from $x^2+y^2=20^2$

$12^2+y^2=400$

$y^2=400-144$

$y^2=256$

$y=\pm\sqrt{256}$

$=\pm\sqrt{16^2}$

$=\pm 16$

use $y=16$.

⑥ $2\cdot 12\cdot 5 + 2\cdot 16\frac{dy}{dt} = 0$

$32\frac{dy}{dt} = -120$

$\frac{dy}{dt} = -\frac{120}{32} = -\frac{8\cdot 15}{8\cdot 4} = -\frac{15}{4}$

sanity check, length is decreasing.

The top of the ladder is sliding down the wall @ $\frac{15}{4}$ feet/second.

3. Find the derivative of

$$f(x) = \sqrt{\frac{x+1}{2x+3}}$$

$$f(x) = \left(\frac{x+1}{2x+3}\right)^{\frac{1}{2}}$$

general power rule / chain

$$f'(x) = \frac{1}{2}\left(\frac{x+1}{2x+3}\right)^{-\frac{1}{2}} \cdot \underbrace{\frac{d}{dx}\left[\frac{x+1}{2x+3}\right]}_{\text{quotient rule}}$$

$$= \frac{1}{2}\left(\frac{x+1}{2x+3}\right)^{-\frac{1}{2}} \cdot \frac{(2x+3)\frac{d}{dx}[x+1] - (x+1)\frac{d}{dx}[2x+3]}{(2x+3)^2}$$

$$= \frac{1}{2}\left(\frac{x+1}{2x+3}\right)^{-\frac{1}{2}} \cdot \frac{(2x+3)\cdot 1 - (x+1)2}{(2x+3)^2}$$

$$= \frac{1}{2} \frac{1}{\left(\frac{x+1}{2x+3}\right)^{\frac{1}{2}}} \quad \frac{2x-2x+3-2}{(2x+3)^2}$$

$$= \frac{1}{2} \cdot \frac{1}{\frac{(x+1)^{\frac{1}{2}}}{(2x+3)^{\frac{1}{2}}}} \cdot \frac{1}{(2x+3)^2}$$

$$= \frac{1}{2} \frac{(2x+3)^{\frac{1}{2}}}{(x+1)^{\frac{1}{2}}} \cdot \frac{1}{(2x+3)^2}$$

$$= \frac{1}{2} \frac{1}{(x+1)^{\frac{1}{2}}(2x+3)^{\frac{3}{2}}} = \frac{1}{2(x+1)^{\frac{1}{2}}(2x+3)^{\frac{3}{2}}}$$

4. Find $\frac{dy}{dx}$ of the equation

chain/general power rule

$$(3x+2y)^{1/3} = xy$$

product rule

$$\frac{d}{dx}\left[(3x+2y)^{\frac{1}{3}}\right] = \frac{d}{dx}[xy]$$

$$\frac{1}{3}(3x+2y)^{-\frac{2}{3}} \cdot \frac{d}{dx}[3x+2y] = \frac{d}{dx}[x]\cdot y + \frac{d}{dx}[y]\cdot x$$

don't forget parenthesis!

$$\frac{1}{3}(3x+2y)^{-\frac{2}{3}} \cdot \left(3+2\frac{dy}{dx}\right) = 1\cdot y + \frac{dy}{dx}\cdot x$$

$$\frac{1}{3}\cdot 3(3x+2y)^{-\frac{2}{3}} + \frac{1}{3}\cdot 2(3x+2y)^{-\frac{2}{3}}\frac{dy}{dx} = y + \frac{dy}{dx}x$$

distributive law

$$\frac{2}{3}(3x+2y)^{-\frac{2}{3}}\frac{dy}{dx} - \frac{dy}{dx}x = y - (3x+2y)^{-\frac{2}{3}}$$

$$\frac{dy}{dx}\left(\frac{2}{3}(3x+2y)^{-\frac{2}{3}} - x\right) = y - (3x+2y)^{-\frac{2}{3}}$$

$$\frac{dy}{dx} = \frac{y-(3x+2y)^{-\frac{2}{3}}}{\frac{2}{3}(3x+2y)^{-\frac{2}{3}} - x} = \frac{y - \frac{1}{(3x+2y)^{\frac{2}{3}}}}{\frac{2}{3(3x+2y)^{\frac{2}{3}}} - x} \cdot \frac{3(3x+2y)^{\frac{2}{3}}}{3(3x+2y)^{\frac{2}{3}}}$$

$$= \frac{3y(3x+2y)^{\frac{2}{3}} - 3}{2 - 3x(3x+2y)^{\frac{2}{3}}}$$

5. A demand equation is

this is $f(p)$

$$x = \frac{1}{5}(225 - p^2)$$

where x is measured in units of a hundred and is the quantity demanded. p is unit price in dollars.

(a) Find the elasticity of demand.

since $f(p) = \frac{1}{5}(225 - p^2) = 45 - \frac{1}{5}p^2$

$f'(p) = -\frac{2}{5}p$

so

only p

$$E(p) = -\frac{p \cdot f'(p)}{f(p)} = -\frac{p \cdot \left(-\frac{2}{5}p\right)}{45 - \frac{1}{5}p^2} = +\frac{2p^2}{5\left(45 - \frac{1}{5}p^2\right)} = \frac{2p^2}{225 - p^2}$$

(b) If the unit price is lowered slightly from \$10, does revenue increase or decrease? Justify with calculations.

$$E(10) = \frac{2 \cdot 10^2}{225 - 10^2} = \frac{2 \cdot 100}{225 - 100} = \frac{200}{125} = \frac{8}{5} > 1$$

So demand is elastic @ $p = \$10$

If the unit price is lowered, we should expect revenue to increase.

(c) When is the demand unitary?

Solve $E(p) = 1$.

$$\frac{2p^2}{225 - p^2} = 1 \to 2p^2 = 225 - p^2 \to 3p^2 = 225 \to p^2 = \frac{225}{3}$$

$$\to p = \pm\sqrt{\frac{225}{3}} \to p = \frac{\sqrt{225}}{\sqrt{3}} = \frac{15}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{15\sqrt{3}}{3} = 5\sqrt{3} \text{ dollars}$$

price is not negative